Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

New and Rare Forms of Fossil Fish from Kansas, by Prof. B. F. Mudge.

Analysis of Clays from different parts of Kansas, by Dr. Wm. H. Saunders.

Additions to the Catalogue of Kansas Plants (continued from last year), by Prof. J. H. Carruth.

Notes from the Zoological Laboratory of the University of Kansas, by Prof. F. H. Snow.

A Mad Stone in Kansas City, by John D. Parker.

A Preliminary List of the Coleoptera of Kansas, by Edwin A. Popenoe.

Observations on the use of the Antennæ of Polyphylla variolosa (Harris), by Prof. F. H. Snow.

The Study of Natural History as a Disciplinary Power, by Prof. W. Wheeler.

The Importance of a Geological Survey of Kansas, by Prof. B. F. Mudge.

Geological Features of the Lower Neosho Valley in Kansas, by W. S. Newlon.

On Determining the Solubilities of Metallic Salts, by Prof. G. E. Patrick.

Disappearance of the Prairie Dog from Kansas, by Prof. B. F. Mudge.

Suggestions on the True Theory of Number, by Prof. F. W. Bardwell.

### OFFICERS.

The following officers were elected for the current year:

President—Frank H. Snow, Lawrence.

Vice-Presidents—B. F. Mudge, Manhattan; John D. Parker, Wyandotte.

Secretary—John Wherrell, Leavenworth.

Treasurer—Robert J. Brown, Leavenworth.

Curators—Frank H. Snow, Lawrence; Edwin A. Popenoe, Topeka; W. K. Kedzie, Manhattan.

### COMMISSIONS.

Geology — H. C. Hovey, Kansas City; B. F. Mudge, Manhattan.

Entomology — Frank H. Snow, Lawrence; Edwin A. Popenoe, Topeka.

Language — D. H. Robinson, Lawrence; J. H. Lee, Manhattan.

Ethnology — H. B. Norton, Emporia.

Engineering — F. W. Bardwell, Lawrence.

Meteorology — John D. Parker, Wyandotte.

Botany — J. H. Carruth, Lawrence; John Wherrell, Leavenworth.

Mineralogy — W. K. Kedzie, Manhattan.

Chemistry — G. E. Patrick, Lawrence.

Mammalia — M. V. B. Knox, Baldwin City.

The Society adjourned to meet at Topeka — the time of meeting to announce.

## SUGGESTIONS OF THE TRUE THEORY OF NUMBER.

BY F. W. BARDWELL.

No one will question the importance of a correct theory of number, though many may be surprised at the suggestion that the current theories are imperfect in any important feature.

It is however true that there is a widespread dissatisfaction with the books which treat of the elements of number; that there are frequent changes in the text-books used in the schools; and that educators, and the friends of education, often complain of the small results of the great amount of time and effort expended in the department of mathematics, and especially given to arithmetic.

These facts justify at least an inquiry in regard to the fundamental theory of number on which methods of instruction properly rest.

### WHAT, THEN, IS NUMBER?

If, as some allege, no formal and perfect definition can be given, yet by means of careful study we should be able to talk intelligently about it, and be able to say confidently, *"this" is number, or "that" is not number.*

Mill says in his System of Logic: "The fact asserted in the definition of number is a physical fact. Each of the numbers two, three, four, etc., denotes physical phenomena and connotes a physical property of those phenomena. Two, for instance, denotes all pairs of things, twelve all dozens of things, connoting what makes them pairs or dozens; and that which makes them so is something physical, since it cannot be denied that two apples are physically distinguishable from three apples, two horses from one horse, and so forth; that they are a different, visible, and tangible phenomena."

### NUMBER USED TO DESIGNATE HOW MANY.

Perhaps it will be admitted that one of the first and most obvious uses of number is to distinguish or define *how many* things are thought of. It may or may not be in answer to a question. In either case, the term called *number*, completely fulfills the function of telling *how many.*

Does number do anything more than tell how many? It appears not. If not, let number be defined as *that which expresses how many things are thought of*, and let this definition be accepted until some defect be found, or until some better definition be invented.

### TWO CLASSES OF NUMBERS.

If there be a group of oranges, consisting of three whole ones and a half of one, that is, one of two equal parts of a whole one, then to describe how many are in this group one would say *three-and-a-half*, and this term obviously performs the same kind of office which in other instances is performed by the term *three*, or by *four*; in other words, it completely fulfills the office of number, and should be included under the name. The same may evidently be affirmed of *one-half* or *three-fourths*. Either may be used to express how many, as one may say "I have ten dollars," while another states "I have *three-fourths* of a dollar," or still another says "I have *one-half* a dollar." It appears then that number may be affirmed of *whole things* or *parts of things*, and this suggests two classes of numbers, one de noting whole things, the other denoting parts of things, called respectively integral and fractional numbers.

To many, no doubt much of the foregoing seems so obvious as scarcely to merit a statement, yet recently an advanced college class in mathematics was puzzled by the question, "is one-half a number?" and the opinions were divided, though a majority thought *not*. The same question propounded to a large convention of teachers, caused a nearly equal division in the opinions expressed, while afterwards an able mathematician, widely known and admitted to be such, expressed serious doubt in regard to the matter, though finally admitting the view here expressed.

The facts just stated, indicate not so much a superficial acquaintance with the subject, as the imperfection of the current methods of presenting the elements of numbers, which should leave such confused notions in the mind, even after subsequent progress had made the problems of the higher mathematics quite familiar. One objected to the existence of fractional number, because " there cannot be such things as three-and-a-half men, or four-and-one fourth sheep." To this it may be answered, that fact simply indicates a feature in the constitution of men and of living beings, which does not permit the existence of life in fractional parts, but in no wise does it impair the flexibility of number which is applicable either to wholes or parts.

In this connection, a practical suggestion occurs, which seems worthy of consideration. Since integral and fractional numbers are two classes of number in general, the exposition of the elementary operations should be applicable alike to both these classes; for instance, the definitions of multiplication and of division should be such as to apply to either integral or fractional numbers. By such means it is believed that the subject of fractional numbers may be divested of much of the difficulty usually encountered by the teacher as well as the pupil.

### DISTINCTION BETWEEN NUMBER AND QUANTITY.

Much confusion seems to exist in the minds of many in regard to the notions of number and quantity. This is partly due to the intimate relation of the things, and partly due to the failure of mathematical authors and writers to make the proper discrimination.

The definition of number is often stated to be "the ratio of two magnitudes or quantities," or, it is said, "number is a unit or collection of units of which one is any magnitude assumed as a standard of measure." In making such statements, authors appear to make number depend upon quantity for its existence, and indeed it is sometimes said, quantity includes both *space and number;* from which it would follow that number is simply one of the kinds of quantity. Prof. Olney says "quantity is the amount or extent of that which may be measured." It seems, however, a better statement to say, quantity expresses the limit of magnitude or extent ; in other words, *quantity expresses "how much."*

The difference between number and quantity is precisely of the same kind as that between the notions of *much* and *many.*

It is noticeable that in the form of words, the comparatives of much and many are identical. Thus we say *much, more, most,* and we say, too, *many, more, most;* but the notions of more and most in one case differ from those in the other case in the same way that much differs from many; that is, in the first case more and most relate to quantity, while in the second case they relate to number.

This use of one word to express two ideas or notions, which though related are yet distinct, indicates some confusion in the minds of those whose use of words has given form to our language; but this fact scarcely excuses at the present day any failure on the part of authors and instructors to make the proper discrimination.

In the French and German languages the distinction is less marked than in the English; and so in the Latin, *multus* and *multi* (much and many) differ only in singular and plural forms of the same word. The same is true of the Greek.

### NUMBER USED IN THE MEASURE OF QUANTITY.

It follows from the foregoing that it is incorrect to say "number is quantity," though number is often used in *measuring* quantity. It is, in fact, seldom that a quantity is measured in any other way than by meaning the *number* of standard units of quantity contained in the quantity to be measured. For instance, four feet, twelve bushels, twenty acres, illustrate this statement. Yet this fact, though it indicates an important use of number, still does not constitute the basis of a definition, and should not be so misused.

### IS MATHEMATICS THE "SCIENCE OF QUANTITY?"

This statement, which has become stereotyped, fails to perform well the office of a definition, even if it does not fail to express the literal truth. The vague use of the term "*quantity*" unfits it to appear as the essential part of a definition. In algebra the current use of this term makes it almost equivalent to the term number, or *symbol of number.* It should be remembered that all algebraic operations are upon number. Thus there can be no multiplication of quantity by quantity; strictly speaking, all multiplication is *purely* by number ; and it would seem better in a science, one of whose chief excellences is *precision*, to call things by their right names. It is believed that a careful consideration will convince any one that all the symbols of algebra are either the symbols of number or of the relations *of* and operations *upon* number. In the application of algebraic principles to problems of quantity, we deal only with the *measure* of quantity in the guise of number. In the elements of geometry, the symbols of magnitude are presented for direct consideration; but in the higher geometry the investigations are made chiefly by algebraic methods; that is, by using the symbols of number to represent the measures of quantities. The statement that mathematics is the science of quantity seems then to be an incomplete definition, if in any true sense it can be called a definition.

### ABSTRACT AND CONCRETE NUMBERS.

Perhaps the frequent occurrence of these terms forms a sufficient reason for noticing them in this connection. Number in its very nature is abstract, and it is an obvious misnomer to call it *concrete* in any case. Two means *two things*, and though the kind of things is not explicitly named, the essential character of number is not affected. The statement "two and three are five," means merely "two things and three things counted together are five things." It may be mentioned in passing, that since the office of number is exclusively that of fixing a limit of "how many," the greater or less extent of "how many," does not at all affect the character of the number applicable to the particular case. Thus the number two or three, possesses as completely the character of number as *a thousand* or *a million*. It follows then that *zero* or *naught*, when used, as it often is, to designate the limit of "how many," is properly included under the name number. It is believed that this consideration is not only logically consistent, but that it simplifies the use of *naught*, as well in arithmetic as in algebra, where it often appears as the root of an equation, and in many instances performs as conspicuous and important services as are assigned to any number.

The necessarily brief limits of the present paper do not permit a further examination of these questions at this time; but it is believed enough has been said to show that a thorough revision of the fundamental theory of number is required in order that this branch of instruction should not fall behind in the developments of the day.